

Calculs de DL

Fonctions trigonométriques

$f(x)$	a	n	DL
$x/\sin x$	0	4	$1 + x^2/6 + 7x^4/360$
$1/\cos x$	0	4	$1 + x^2/2 + 5x^4/24$
$\ln(\sin x/x)$	0	6	$-x^2/6 - x^4/180 - x^6/2835$
$\exp(\sin x/x)$	0	4	$e(1 - x^2/6 + x^4/45)$
$\sqrt{\tan x}$	$\pi/4$	2	$1 + h + h^2/2$
$\sin(x + x^2 + x^3 - x^4)$	0	4	$x + x^2 + 5x^3/6 - 3x^4/2$
$\ln(x \tan(1/x))$	∞	4	$x^{-2}/3 + 7x^{-4}/90$
$(1 - \cos x)/(e^x - 1)^2$	0	2	$1/2 - x/2 + x^2/6$
$\sin((\pi \cos x)/2)$	0	6	$1 - \pi^2 x^4/32 + \pi^2 x^6/192$
$\cos x \ln(1 + x)$	0	4	$x - x^2/2 - x^3/6$
$(\sin x - 1)/(\cos x + 1)$	0	2	$-1/2 + x/2 - x^2/8$
$\ln(2 \cos x + \tan x)$	0	4	$\ln 2 + x/2 - 5x^2/8 + 11x^3/24 - 59x^4/192$
$e^{\cos x}$	0	5	$e(1 - x^2/2 + x^4/6)$

Fonctions circulaires inverses

$f(x)$	a	n	DL
$\arcsin^2 x$	0	6	$x^2 + x^4/3 + 8x^6/45$
$1/\arcsin^2 x$	0	2	$x^{-2} - 1/3 - x^2/15$
$\arctan \sqrt{(x+1)/(x+2)}$	∞	2	$\pi/4 - x^{-1}/4 + 3x^{-2}/8$
$\arccos(\sin x/x)$	0	3	$ x /\sqrt{3}(1 - x^2/90)$
$1/\arctan x$	0	5	$x^{-1} + x/3 - 4x^3/45 + 44x^5/945$
$\arcsin \sqrt{x}$	$1/4$	3	$\pi/6 + 1/\sqrt{3}(2h - 4h^2/3 + 32h^3/9)$
$\arcsin(\sin^2 x)$	0	8	$x^2 - x^4/3 + 19x^6/90 - 107x^8/630$
$\arctan(1 + x)$	0	4	$\pi/4 + x/2 - x^2/4 + x^3/12$
$\arcsin x/(x - x^2)$	0	2	$1 + x + 7x^2/6$
$e^{\arcsin x}$	$1/2$	2	$e^{\pi/6}(1 + 2h/\sqrt{3} + 2(1 + \sqrt{3})h^2/(3\sqrt{3}))$
$e^{1/x} \arctan x$	∞	3	$\frac{\pi}{2} + (\frac{\pi}{2} - 1)x^{-1} + (\frac{\pi}{4} - 1)x^{-2} + (\frac{\pi}{12} - \frac{1}{6})x^{-3}$

Exponentielle et logarithme

$f(x)$	a	n	DL
$x/(e^x - 1)$	0	2	$1 - x/2 + x^2/12$
$\ln x/\sqrt{x}$	1	3	$h - h^2 + 23h^3/24$
$\ln((2-x)/(3-x^2))$	0	2	$\ln(2/3) - x/2 + 5x^2/24$
$\ln(1+x)/(1-x+x^2)$	0	3	$x + x^2/2 - x^3/6$
$x/\ln(1+x)$	0	1	$x^{-1} + 1/2 + 5x/12$
$\ln(\ln(1+x)/x)$	0	3	$-x/2 + 5x^2/24 - x^3/8$
$\ln(a^x + b^x)$	0	2	$\ln 2 + x \ln \sqrt{ab} + x^2 \ln^2(a/b)/8$
$\exp(1/x)/x^2$	1	3	$e(1 - 3h + 13h^2/2 - 73h^3/6)$

Fonctions hyperboliques inverses

$f(x)$	a	n	DL
$\operatorname{argth}(\sin x)$	0	5	$x + x^3/6 + x^5/24$
$\operatorname{argsh}(e^x)$	0	2	$\ln(1 + \sqrt{2}) + 1/\sqrt{2}(x + x^2/4)$

Formes exponentielles

$f(x)$	a	n	DL
$(1 - x + x^2)^{1/x}$	0	2	$e^{-1}(1 + x/2 + 19x^2/24)$
$((1 + x)/(1 - x))^\alpha$	0	3	$1 + 2\alpha x + 2\alpha^2 x^2 + 2\alpha(2\alpha^2 + 1)x^3/3$
$(\sin x/x)^{2/x^2}$	0	3	$e^{-1/3}(1 - x^2/90)$
$(\sin x/x)^{3/x^2}$	0	4	$e^{-1/2}(1 - x^2/60 - 139x^4/151200)$
$(1 + \sin x)^{1/x}$	0	2	$e(1 - x/2 + 7x^2/24)$
$(1 + \sin x + \cos x)^x$	0	2	$1 + x \ln 2 + x^2(\ln^2 2 + 1)/2$
$(\sin x)^{\sin x}$	$\pi/2$	4	$1 - h^2/2 + 7h^4/24$
$(\tan x)^{\tan 2x}$	$\pi/4$	4	$e^{-1}(1 + 2h^2/3 + 4h^4/5)$

Radicaux

$f(x)$	a	n	DL
$x\sqrt{(x-1)/(x+1)}$	2	3	$1/\sqrt{3}(2 + 5h/3 + h^3/54)$
$\sqrt{1 + \sqrt{1-x}}$	0	3	$\sqrt{2}(1 - x/8 - 5x^2/128 - 21x^3/1024)$
$\sqrt{1 - \sqrt{1-x^2}}$	0	5	$ x /\sqrt{2}(1 + x^2/8 + 7x^4/128)$
$e^x - \sqrt{1+2x}$	0	5	$x^2 - x^3/3 + 2x^4/3 - 13x^5/15$
$(3x^3 + x^2 + 3x^3 - x^2)/x$	∞	3	$2 - 2x^{-2}/9$

Exercice 1 EIT 1999

Calculer le développement limité de $\left(\frac{\tan x}{x}\right)^{1/x^2}$ en 0 à l'ordre 3.

EIT 1999

Solution 1 $e^{1/3} \left(1 + \frac{7}{90}x^2 + o(x^3)\right)$.